Introduction to Manifolds

Assignment 5
Due Date: 30/10/2017

Problem 1: Let M be a smooth n-manifold. Prove that the tangent bundle TM is Hausdorff.

Problem 2: Let M be a smooth n-manifold and q be a point in M and U be any neighborhood of q. Construct a smooth bump function at q supported in U.
(You may explicitly use the bump function at $0 \in \mathbb{R}^n$ that was constructed in class.)

Problem 3: Let $x_1, y_1, \ldots, x_n, y_n$ be the standard coordinates on $\mathbb{R}^{2n}$. Show that

$$X = \sum_{i=1}^{n} x_i \frac{\partial}{\partial y_i} - y_i \frac{\partial}{\partial x_i}$$

is a nowhere-vanishing vector field on the unit sphere S^{2n-1}.

Problem 4: Let X be the vector field $\dfrac{d}{dx}$ on the punctured line $\mathbb{R} - \{0\}$. Find the maximal integral curve of X starting at $x = 1$.