Introduction to Manifolds

Assignment 4
Due Date: 21/09/2017

Problem 1: Show that the map $f : \mathbb{R}/\mathbb{Z} \to S^1$ defined by $x \mapsto (\cos 2\pi x, \sin 2\pi x)$ is a diffeomorphism.

Problem 2: Find all points in $\mathbb{R}^3$ in a neighborhood of which the functions $x, x^2 + y^2 + z^2 - 1, z$ can serve as a local coordinate system.

Problem 3: For $\theta \in [0, 2\pi)$, let $F^\theta : \mathbb{R}^2 \to \mathbb{R}^2$ be the rotation through θ (in counterclockwise direction) map. Given $p \in \mathbb{R}^2$ find a matrix representing the differential $F^\theta_{*,p} : T_p\mathbb{R}^2 \to T_p\mathbb{R}^2$.

Problem 4: Consider the map $f : S^2 \to \mathbb{R}$ given by $(x, y, z) \mapsto z$. Find all the critical points of f.

Problem 5: Let $n \in \mathbb{Z}$ and let $g_n : S^1 \to S^1$ be given by $z \mapsto z^n$ (here $S^1 \subset \mathbb{C}$). Compute the differential of g_n at some point $w \in S^1$.