Topics in Combinatorics

Assignment 2
Due Date: 22/01/2018

Problem 1: Let $\mathscr{A}$ be an arrangement of hyperplanes in $\mathbb{R}^n$. Then prove that the intersection poset $L(\mathscr{A})$ is graded of rank equal to $\text{rank}(\mathscr{A})$. The rank function is given by the codimension of the corresponding intersection.

Problem 2: Prove that

$$L(\text{ess}(\mathscr{A})) \cong L(\mathscr{A}).$$

Problem 3: Let $\mathscr{A}$ be the coordinate (boolean) arrangement in $\mathbb{R}^n$. Prove that $L(\mathscr{A})$ is isomorphic to the poset of all subsets of $[n]$ ordered by inclusion. Compute the characteristic polynomial of this arrangement.

Problem 4: Determine the characteristic polynomial of arrangement of n lines in general position. Using this polynomial find $r(\mathscr{A})$ and $b(\mathscr{A})$.

Problem 5: For an arrangement $\mathscr{A}$ and its essentialization $\text{ess}(\mathscr{A})$ show that

$$t^{\text{rank}(\mathscr{A})}\chi_{\mathscr{A}}(t) = t^{\dim \mathscr{A}}\chi_{\text{ess}(\mathscr{A})}(t).$$

Moreover, if $\chi_{\mathscr{A}}(t)$ is divisible by t^k but not t^{k+1} then show that $\text{rank}(\mathscr{A}) = n - k$.