# COMBINATORICS 1
# ASSIGNMENT 4
# (DUE DATE: 10/10/2016)

---

- All posets are finite and graded. For a poset $P$ the symbol $\hat{P}$ stands $P$ adjoined with $\hat{0}, \hat{1}$.
- The symbol $\mathcal{I}(P)$ stands for the set of all closed intervals in $P$
- The Möbius polynomial of $P$ is defined as

$$\mathcal{M}(P, x) := \sum_{[s,t] \in \mathcal{I}(P)} \mu(s,t) x^{\mathrm{rank}(t) - \mathrm{rank}(s)}.$$

- The characteristic polynomial of a hyperplane arrangement $\mathcal{A}$ is defined as

$$\chi(\mathcal{A}, x) := \sum_{s \in L(\mathcal{A})} \mu(s) x^{\dim s}.$$

---

(1) (10 points) If $P$ has either $\hat{0}$ or $\hat{1}$ then prove that $\mathcal{M}(P, 1) = 1$.

(2) (5 points) Further conclude that for an arbitrary poset $P$

$$\mathcal{M}(P, 1) = \mu_{\hat{P}}(\hat{0}, \hat{1}) + 1.$$

(3) (5 points) Let $\mathcal{A}$ be an arrangement of hyperplanes in $\mathbb{R}^n$. Then prove the following

$$x^{\dim(\mathrm{ess}(\mathcal{A}))} \chi(\mathcal{A}, x) = x^{\dim(\mathcal{A})} \chi(\mathrm{ess}(\mathcal{A}), x).$$

(4) (10 points) Compute the number of connected components of the complement of the braid arrangement $A_{n-1}$ without using the Zaslavsky's formula.

(5) (10 points) Prove that the intersection lattice of the braid arrangement $A_{n-1}$ is isomorphic to the lattice of partitions of $[n]$.

(6) (10 points) Compute the characteristic polynomial of $A_{n-1}$.