COMBINATORICS 1
ASSIGNMENT 1
(DUE DATE: 11/08/2016)

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(1) (10 points) Let $A_1, \ldots, A_m$ be subsets of a finite set A. Let $N_0 := |A|$ and for $k > 0$, let

$$N_k := \sum_{\substack{|I|=k \\ I \subseteq [m]}} |A_I|,$$

where $A_I := \bigcap_{i \in I} A_i$. Then prove using the Principle of Inclusion and Exclusion that the number of elements of A that are in none of the A_i's is

$$\sum_{i=0}^{m} (-1)^i N_i.$$

Conclude further that

$$| \bigcup_i A_i | = \sum_{i=1}^{m} (-1)^{i-1} N_i. \tag{1}$$

(2) (5 points) Prove the following identity

$$\prod_{i=1}^{m} (1 - x_i) = \sum_{I \subseteq [m]} (-1)^{|I|} \prod_{i \in I} x_i.$$

(3) (5 points) Prove Equation 1 using induction on m

(4) (10 points) List (draw) all the combinatorially distinct arrangements of 4 lines. Why is your list complete?

(5) (10 points) Let $n \geq 3$ denote the number of straight lines in the plane and p be the maximal number of parallel lines. Then prove that

$$f_2 \geq (p + 1)(n - p + 1).$$

Further conclude that f_2 can not take any value in the interval $(n + 1, 2n)$.