Model-Checking Event Structures

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Logics and verification

Temporal logic

- Used to describe properties to be verified
- Comes in two basic flavours

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Branching-time temporal logic

- Collect all runs of the system in a single structure, the computation tree
- Interpret formulas over computation tree
- Can quantify over runs, compare runs, ...

The Computation Tree

- We will work finite-state systems
- Start at an initial state and explore all executions
- Unfold the system into a tree

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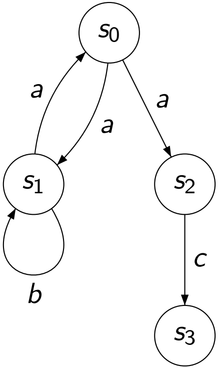

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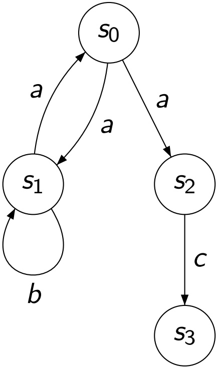

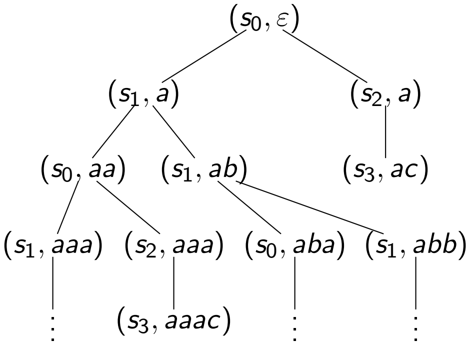

Concurrent systems

- Suppose the system is a collection of interacting components
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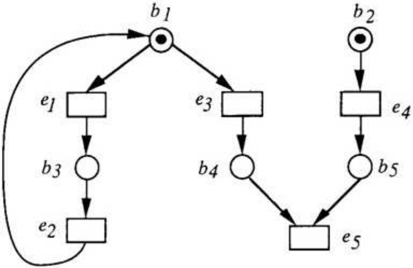

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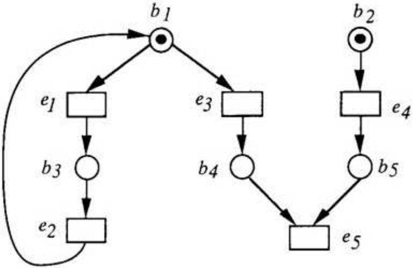

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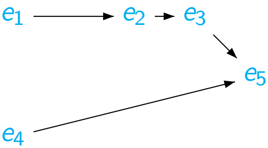
Runs as partial orders

- Convenient to view each execution as a labelled partial order
- Actions can be related in two ways
  - Causality
    An occurrence of b causally depends on an occurrence of a if a must happen before b happens
  - Concurrency
    An occurrence of b is independent of an occurrence of a if they can occur in any order



Runs as partial orders

- In many interesting cases, the runs of concurrent systems can be described as traces [Mazurkiewicz]
- Actions are enriched with independence relation specifying which pairs are independent
  - Symmetric, irreflexive
  - Typically derived from structure of underlying system
    - Actions performed by disjoint sets of components
- In a linearization, adjacent independent actions can be swapped to yield an equivalent linearization

$e_1 \; e_2 \; e_3 \; e_4 \; e_5$

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Temporal logics for concurrent systems

- Temporal logic interpreted on linearizations of runs makes too many distinctions
  - A property such as e_2 is immediately followed by e_4 is true in some linearizations and not in others
- Modify temporal logic to express causality and concurrency
- What about linear-time vs branching-time?
- How do we represent the computation tree of a concurrent system?

Computation tree of a concurrent system

- In sequential systems, computation tree glues together all runs in a single branching structure
- In concurrent systems, each run is a labelled partial order (a trace)
- Need to glue together traces to form a tree

Event structures

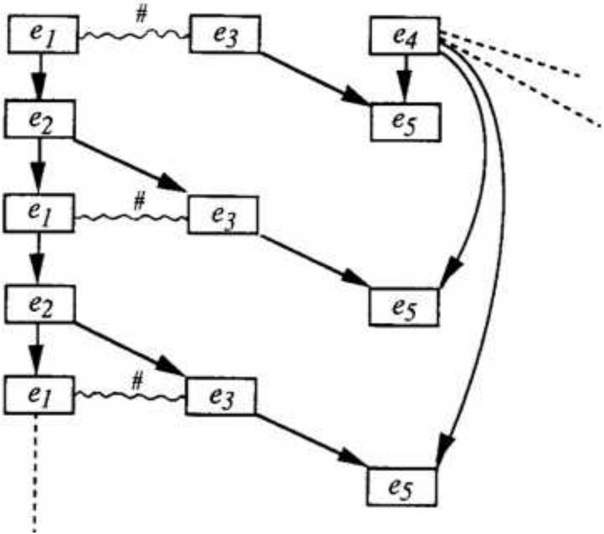
- Each action occurs in a context — what has happened earlier
- Each different occurrence of an action is an event
- In a single trace, events are related by causality or concurrency
- Across traces, events are related by conflict
  - Choosing between two mutually incompatible events generates different runs

Event Structures . . .

Formally, an event structure is of the form $ES = (E, \leq, \#)$

- E is the set of events
- $\leq$ is the causality relation (a partial order)
- $\#$ is a binary conflict relation
  - Irreflexive, symmetric
- Conflict is inherited via causality
  - $e \# f$ and $f \leq f'$ implies $e \# f'$
- Two events are concurrent if they are not related by $\leq$ or $\#$ — $e \; co \; f$

Event Structures . . .



Event Structures . . .

- A configuration of an event structure is a set $X \subseteq E$ such that
  - X is $\downarrow$-closed
  - X is conflict free
- A configuration represents a set of events that have happened so far — a global state of the system

From traces to event structures

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- t and t' are compatible if there is t'' such that $t \leq t''$ and $t' \leq t''$
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- $t \# t'$ if t and t' are not compatible

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- $t \# t'$ if t and t' are not compatible
- Identify events with prime traces
  - Prime trace: Only one maximal element
  - "Earliest" occurrence of an action

Temporal logics for event structures

- Interpret formulas at events of an event structure
  - Event e denotes the minimal configuration $\downarrow e$ where it occurs
  - $\downarrow e$ can be thought of as the local state of the components involved in e
- Modalities to express causality, conflict, concurrency
  - $ES, e \models A_{\leq}\varphi$ if at every f such that $e \leq f$, $ES, f \models \varphi$
  - $ES, e \models E_{\leq}\varphi$ if there exists f such that $e \leq f$ and $ES, f \models \varphi$
  - $ES, e \models A_{\#}\varphi$ if at every f such that $e \# f$, $ES, f \models \varphi$
  - $ES, e \models E_{\#}\varphi$ if there exists f such that $e \# f$ and $ES, f \models \varphi$
  - $ES, e \models A_{co}\varphi$ if at every f such that $e \; co \; f$, $ES, f \models \varphi$
  - $ES, e \models E_{co}\varphi$ if there exists f such that $e \; co \; f$ and $ES, f \models \varphi$

$\lessdot$ and $\#_\mu$

The immediate successor relation $\lessdot$ generates $\leq$

$e \lessdot f$ if $e \leq f$ and for all g, $e \leq g \leq f \Rightarrow e = g$ or $g = f$

Corresponding modality

- $ES, e \models A_{\lessdot}\varphi$ if at every f such that $e \lessdot f$, $ES, f \models \varphi$
- $ES, e \models E_{\lessdot}\varphi$ if there exists f such that $e \lessdot f$ and $ES, f \models \varphi$

Minimal conflict relation $\#_\mu$ generates $\#$ via $\leq$

$e\#_\mu f$ if $e\#f$ and for all $e' \lessdot e$, $f' \lessdot f$, it is not the case that $e\#f'$ or $e'\#f$

Corresponding modality

- $ES, e \models A_{\#_\mu}\varphi$ if at every f such that $e\#_\mu f$, $ES, f \models \varphi$
- $ES, e \models E_{\#_\mu}\varphi$ if there exists f such that $e\#_\mu f$ and $ES, f \models \varphi$

The model-checking problem

- Start with a finite-state representation of the system, M
  - Petri net
  - Product of automata
  - ...
- Atomic propositions AP describing properties of (local) states
  - Valuation assigns a subset of AP to each (local) state
- In ES_M, event structure of M, each configuration $\downarrow e$ corresponds to a unique (global) state of M
- Formulas for system properties built from AP, boolean operations, event structure modalities
  - $A_{\leq}(b \Rightarrow E_{\#} a)$
  - $A_{\lessdot}((b \Rightarrow E_{\#_\mu} a) \wedge (a \Rightarrow E_{\#_\mu} b))$
- Does $ES_M, e \models \varphi$?

The model-checking problem

Apparently only two papers addressing this topic.

- Model-Checking for a Subclass of Event Structures
  W Penczek
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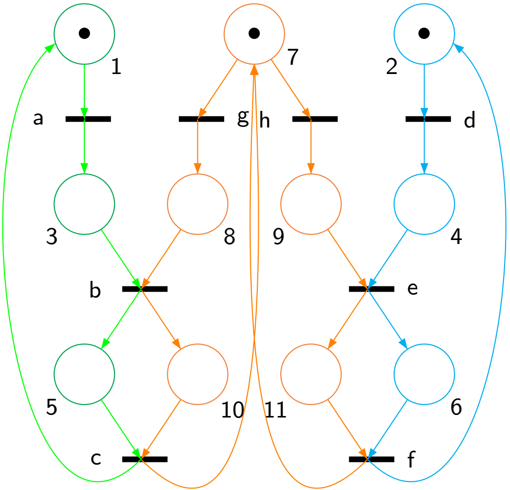
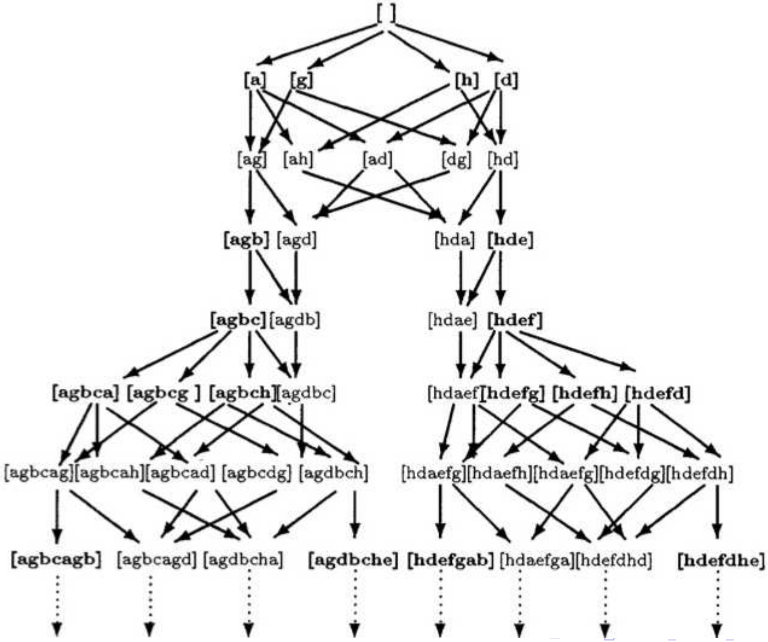
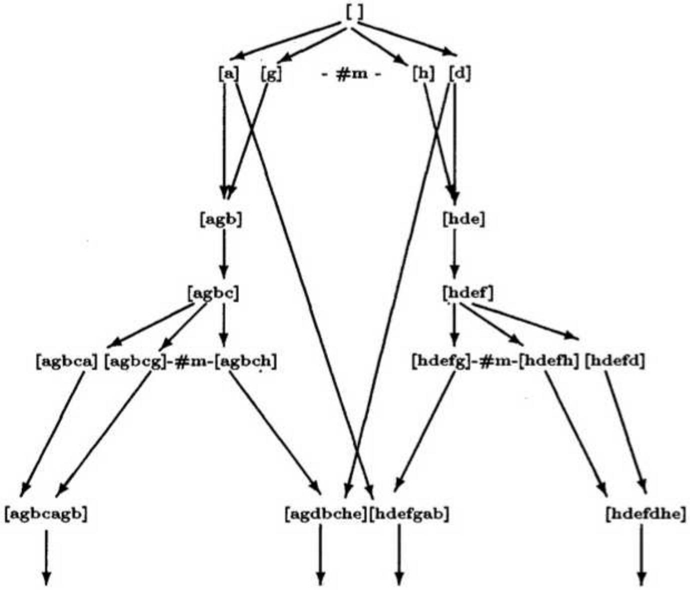
Running example



Trace semantics



Event structure semantics



Penczek's result

- Restrict the modalities
- Only $A_{\leq}/E_{\leq}$, $A_{\lessdot}/E_{\lessdot}$, $A_{\#_m u}/E_{\#_\mu}$
- No A_{co}/E_{co}
- Though $\#$ is generated by $\#_\mu$ and $\leq$, cannot fully express $A_{\#}/E_{\#}$
  - $\# = \leq^{-1} \circ \#_\mu \circ \leq$
  - No past modality to capture $\leq^{-1}$

Quotienting the trace system

First attempt

- Two traces are equivalent if they reach the same global state
- This is not sufficient,
  - $[]$, $[agbc]$ and $[hdef]$ all lead to $(1, 2, 7)$
  - Causal futures are different

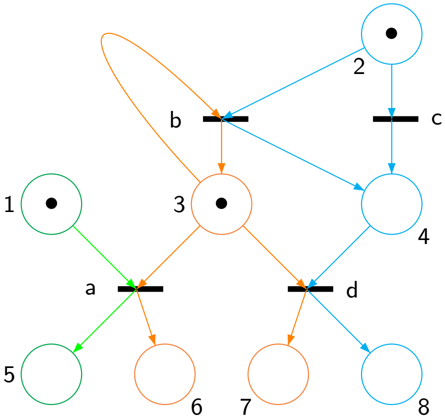
Refine further

- Two traces are equivalent if they reach the same global state and the maximal actions in the traces are the same

Quotienting the trace system . . .

- Recall that events are prime traces
- Let e, f be two events
- Would like the following property
  - If $[e] = [f]$, then $ES_M, e \models \varphi$ iff $ES_M, f \models \varphi$
- This does not hold in general

Another example



Quotienting the trace system ...

- Recall that events are prime traces
- Let e, f be two events
- Would like the following property
  - If $[e] = [f]$, then $ES_M, e \models \varphi$ iff $ES_M, f \models \varphi$
- This does not hold in general
  - In this example, $[bd]$ and $[cd]$ are prime traces reaching same global state $(1, 7, 8)$ with same maximal event d
  - $[a]$ and $[ba]$ are prime traces that reach different global states
  - $[bd]\#_\mu[ba]$ and $[cd]\#_\mu[a]$, so $[bd]$ and $[cd]$ don't satisfy the same $E_{\#_\mu}$ formulas.

Free-choice property

- Restrict systems to have the free-choice property
- Intuitively, choices available to one component cannot change due to actions of another component
  - Second example is not free-choice — if third process executes c, second process loses option of performing b (and vice versa)

Formally

- If $s \xrightarrow{a} s'$ involves components P_a and $t \xrightarrow{b} t'$ involves components P_b then
  - Either $P_a \cap P_b = \emptyset$, or,
  - If $s[j] = t[j]$ for some process j, then $P_a = P_b$ and for all $j \in P_a = P_b$, $s[j] = t[j]$.

Free-choice property . . .

- Free-choice ensure that if $e \#_\mu f'$ then
  - There is a common prefix t such that $e = ta$, $f = tb$
  - Actions a and b involve exactly the same set of components

Recall our definition of a quotiented trace system

$$t \equiv t' \textit{ if } t \textit{ and } t' \textit{ reach the same global state and have the same set of maximal actions}$$

Our desired property

- If $e \equiv f$, then $ES_M, e \models \varphi$ iff $ES_M, f \models \varphi$

can be proved by structural induction on φ from the following

- If $e \equiv f$ and $e \lessdot e'$, there exists f' such that $f \lessdot f'$ and $e' \equiv f'$
- If $e \equiv f$ and $e \#_\mu e'$, there exists f' such that $f \#_\mu f'$ and $e' \equiv f'$

The model-checking algorithm

Naive

- Compute the quotiented trace system for the given model
  - This is a finite object
- Identify prime traces as events and constructed corresponding "quotiented" event structure
- Decompose formulas and use CTL-style bottom-up labelling to identify the formulas true at each event

More efficient

- Use partial order techniques to construct a representative subset of the quotiented trace system
- Translate event structure logic to CTL and direcly use CTL model checking on the quotiented trace system
  - Use a special proposition to mark prime traces

Beyond Penczek's result

- Can we extend the algorithm to full logic involving A_{co}/E_{co} and $A_{\#}/E_{\#}$?
- Can the free-choice restriction be relaxed?