Applications of learning theory in verification

Madhavan Mukund

Chennai Mathematical Institute
http://www.cmi.ac.in/~madhavan

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(Adapted from material contributed by P Madhusudan)

Motivation

- Abstraction is an important tool in verification
  - Build a coarse model M from a system description S
  - Every run of S is also a run of M
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- Other problems in verification can also benefit from this approach

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- Two verification problems
  - Compositional verification of $P \parallel Q$
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- Two verification problems
  - Compositional verification of $P \parallel Q$
  - Deriving interface specification for a module
- Learning regular languages
  - Active learner model [Angluin'86]
  - A tutorial introduction to the learning algorithm
- How to apply learning for the two problems above
- Some pointers to other applications

Compositional verification

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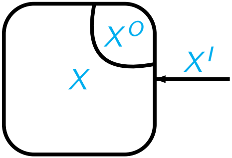
- Does $P \parallel Q$ satisfy a safety specification φ?
- Assume guarantee reasoning
  - Find R such that:
    - $P \parallel R \models \varphi$
    - Behaviours of Q are included in behaviours of R
  - R may be small compared to P and Q.

Compositional verification ...

Module P

State variables X

- output variables $X^O \subseteq X$,
  disjoint set of input variables X^I



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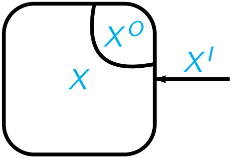

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Compositional verification . . .

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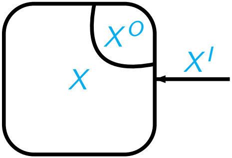


- State variables X
  output variables $X^O \subseteq X$,
  disjoint set of input variables X^I
- Assume we are working with boolean abstraction
- State : $s : (X \uplus X^I) \rightarrow \{0, 1\}$

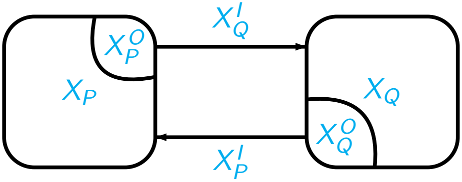
  Transition : $T \subseteq (S \setminus S^I) \times S^I \times (S \setminus S^I)$

  Behaviour : $s_1 s_2 \ldots$

  Visible Behaviour : $s_1^{I \cup O} s_2^{I \cup O} \ldots$

Module composition

$P \parallel Q$: Outputs of P are inputs to Q and vice versa



- $VisBeh(P \parallel Q) = VisBeh(P) \cap VisBeh(Q)$

Compositional verification of modules

Safety property φ: boolean formula over $X^I \cup X^O$

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- Find R such that:
  - $P \parallel R \models \varphi$
  - $\mathit{VisBeh}(Q) \subseteq \mathit{VisBeh}(R)$
- Learn a regular language R with small DFA?

Interface synthesis

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Can we learn a maximal interface?

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Complexity will be measured on the complexity of the language: the minimum number of states needed to capture T.

Active learning [Angluin'86]

- Learner asks questions:
- Membership: Is $w \in T$?
  - Yes or No
- Equivalence question: Is $T = L(C)$?
  - Yes or No+counterexample
  - Counterexample is in $(T \setminus L(C)) \cup (L(C) \setminus T)$.

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Theorem (Angluin, Rivest-Schapire, Kearns-Vazirani)

Regular languages can be learnt using at most $O(kn^2 + n \log m)$ membership and $O(n)$ equivalence queries.

- n — *size of the minimal DFA accepting target language* T
- m — *size of the largest counterexample*
- k — *size of the alphabet.*

Also, in time polynomial in $O(kn^2 + n \log m)$.

How do we learn T?

Key points

- How many states are there?
- How do we reach these states from the initial state?
- How do we build the transitions correctly?

When are states different?

Simple observation:

Let u and v be two strings.

> *If* $\exists w$ *such that* $uw \in T \iff vw \notin T$,
> *then* u *and* v *must lead to different states.*

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If we find n strings $s_1, \ldots, s_n$, that are pairwise distinguishable, we know that automaton for T has (at least) n states.

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Access string to a state q

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An observation pack

Access strings	s_1	s_2	$\dots$	$\dots$	s_k
Experiments	E_{s_1}	E_{s_2}	$\dots$	$\dots$	E_{s_k}

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- Each E_{s_i} consists of a set of pairs of the form $(u, +)$ or $(u, -)$:
  - $(u, +) \in E_{s_i}$ implies $s_i.u_i \in T$
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- $\varepsilon \in S$, and $\varepsilon \in E_{s_i}$ for each i.

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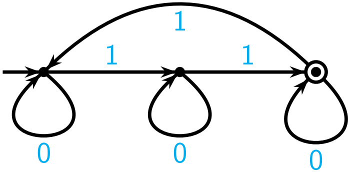
Note: If an observation pack with n access strings exists, then minimal automaton for T has *at least* n states.

Example

Target language T: strings over $\{0, 1\}$ where $\#1's = 2 \bmod 3$



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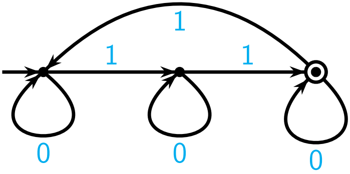

An observation pack:

Access strings	ε	010
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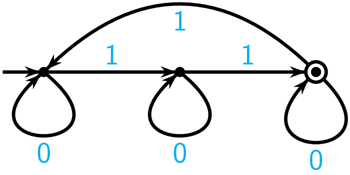

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$\varepsilon.\varepsilon \notin T$; $010.\varepsilon \notin T$
$\varepsilon.10 \notin T$; $010.10 \in T$

Likeness and escape

Let O be an observation pack.

A word w is like an access string s in O, if w agrees with s on all the experiments in E_s.
i.e. ,$\forall u \in E_s$, $wu \in T$ iff $su \in T$.

Note: No two access strings are alike $\Rightarrow w$ can be like *at most one* access string in O, since

If w is *not* like any access string, we say it escapes the pack.

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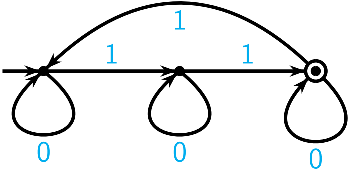

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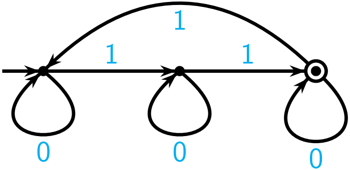

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The word 001 is like 010 (since $001.\varepsilon \notin T$, $001.10 \in T$).

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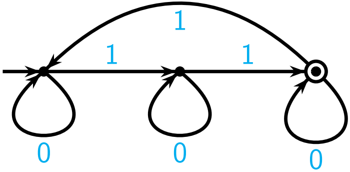

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The word 001 is like 010 (since $001.\varepsilon \notin T$, $001.10 \in T$).

The word 11 is not like any access string in O (since $11.\varepsilon \in T$).
So 11 *escapes*.

Expanding a pack

If O is an observation pack, and w escapes O, then we can expand O to include w:

- Add w as a new access string
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...and has one more access string.

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- Mark a state s final iff $(\varepsilon, +) \in E_s$.

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So, the whole problem reduces to finding an observation pack with n access strings!!

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Equivalence query:

- Build conjecture automaton C.
- Ask teacher "$L(C) = T$?"
- Use counterexample given by teacher to generate new access string.

A learning example. . .

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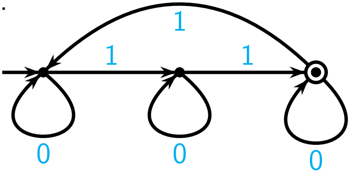

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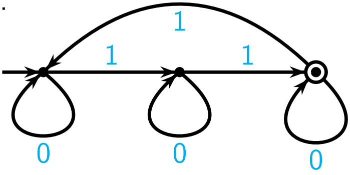

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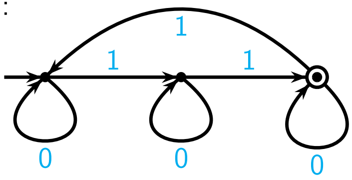

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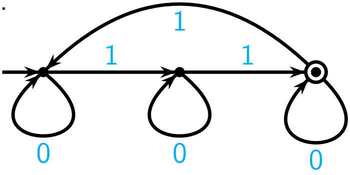
Check closure:

0 is like ε (since $0 \notin T$).

1 is like ε (since $1 \notin T$).

A learning example...

Target language T:



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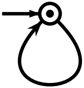

Counter-example: $101 \in T \setminus L(C)$

Run of 101 on C: $s_0 \xrightarrow{1} s_0 \xrightarrow{0} s_0 \xrightarrow{1} s_0$

- $s_0 = \varepsilon$

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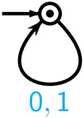

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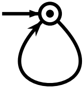

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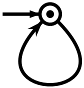

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- $s_0 = \varepsilon$
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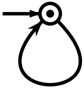
- $s_0.01 \notin T$.
- So we cannot go on 1 to s_0!
  (since 01 distinguishes 1 and s_0)

A learning example . . .



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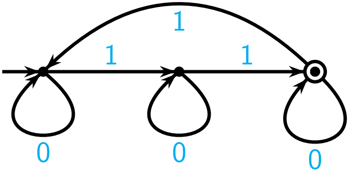
- $s_0.01 \notin T$.
- So we cannot go on 1 to s_0!
  (since 01 distinguishes 1 and s_0)
- So let's add 01 as experiment string for s_0.

A learning example

T



A learning example

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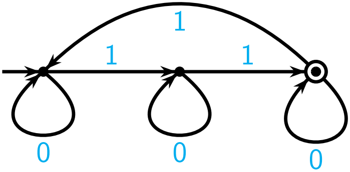

Access strings	$s_0 = \varepsilon$	$s_1 = 1$
Experiments	$(\varepsilon, -)$ $(01, -)$	$(\varepsilon, -)$ $(01, +)$

A learning example

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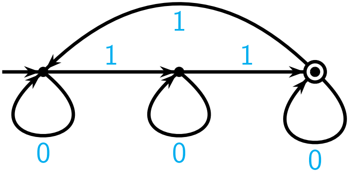

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Check closure:
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A learning example

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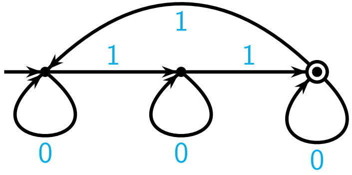

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But 11 is neither like ε nor like 1 (since $11 \in T$).

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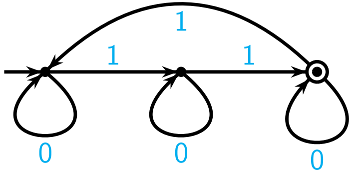

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Check closure:
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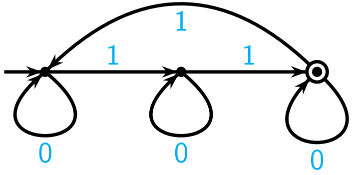
But 11 is neither like ε nor like 1 (since $11 \in T$).

So 11 escapes and forms a new access string.

A learning example ...

T



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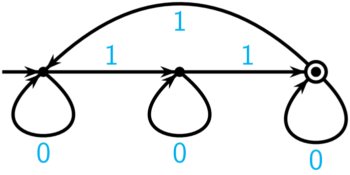

Access strings	$s_0 = \varepsilon$	$s_1 = 1$	$s_2 = 11$
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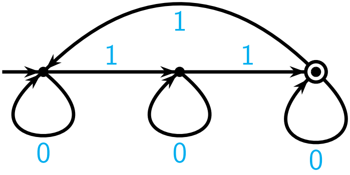

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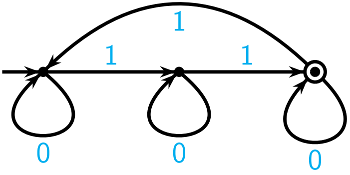
Check closure:
0 is like s_0; 10 is like 1;
110 is like 11; 111 is like 0.

A learning example . . .

T



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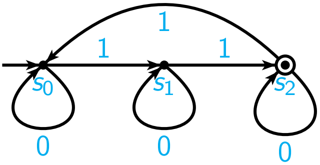

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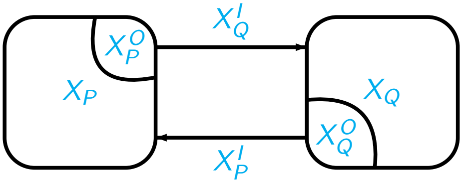
Compositional verification of modules



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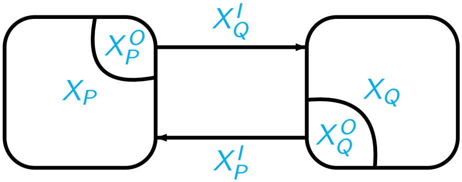

Safety property φ: boolean formula over $X^I \cup X^O$

- $s_1 s_2 \ldots \models \varphi$ if for each i, $s_i^{I \cup O} \models \varphi$

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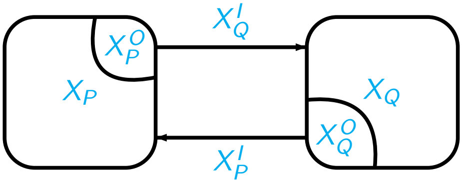

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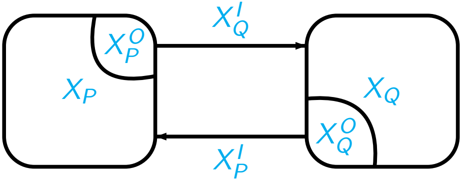

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Assume guarantee reasoning

- Find R such that:
  - $P \parallel R \models \varphi$
  - $VisBeh(Q) \subseteq VisBeh(R)$
- Learn a regular language R with small DFA?

Compositional verification of modules . . .

Most permissive R

$$L_{\mathsf{max}} = \{\sigma \mid \sigma \in \mathit{VisBeh}(P) \Rightarrow \sigma \models \varphi\}$$

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Target language is unknown!

Compositional verification: Implementing the teacher ...

Recall that $L_{\min} \subseteq R \subseteq L_{\max}$

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Practical note: Use BDDs to deal with large alphabet $X^I \cup X^O$

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- An interface I is good if all runs consistent with I satisfy φ

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Given an class C and an interface I, interaction is a game over $C \parallel I$

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